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06MAT41

Fourth Semester B.E. Degree Examination, June/July 2011
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions each from Part – A and Part - B.***

PART – A

1 a. Using Taylor's series method, find y at x = 0.1 and x = 0.2 considering upto 4th degree terms. Given that $\frac{dy}{dx} = x^2y - 1$ and y(0) = 1. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) = 1, find y at x=0.2 using Runge – Kutta method of 4th order taking step – length h = 0.2 accurate upto 4th decimal place. **(07 Marks)**

c. Given that $\frac{dy}{dx} = x^2(1+y)$ and y(1) = 1 ; y(1.1) = 1.233 ; y(1.2) = 1.548 ; y(1.3) = 1.979, find y at x = 1.4 using Adams – Bashforth predictor and corrector formula. **(07 Marks)**

2 a. Find Analytic function whose real part is $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$. **(06 Marks)**

b. Under the transformation $W = e^Z$, prove that family of lines parallel to y – axis in Z – plane transforms into family of concentric circles in W – plane. **(07 Marks)**

c. Find Bilinear transformation, that transforms Z = -1, i, 1 on to points W = 1, i, -1, in W – plane respectively. Also find invariant points. **(07 Marks)**

3 a. Evaluate $\int_C \frac{e^{2Z}}{(Z+1)(Z+2)} dZ$, where 'C' is a circle | Z | = 3. **(06 Marks)**

b. Obtain the power series which represents the function $f(Z) = \frac{Z^2 - 1}{Z^2 + 5Z + 6}$ in the region $2 < |Z| < 3$. **(07 Marks)**

c. Using Cauchy's Residue theorem evaluate $\int_C \frac{2Z^2 + 1}{(Z+1)^2(Z-2)} dZ$, where 'C' is circle with | Z | = 3. **(07 Marks)**

4 a. Using Frobenius series solution method, solve $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Reduce the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (k^2x^2 - n^2)\, y = 0$ into Bessel's form and write the complete solutions for n is not integral or zero. **(07 Marks)**

c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre's polynomial. **(07 Marks)**

PART – B

5 a. Fit the best possible curve of the form $y = a + bx$, using method of Least square for the data: **(06 Marks)**

X :	1	3	4	6	8	9	11	14
Y :	1	2	4	4	5	7	8	9

b. The lines of regressions are $x + 2y = 5$ and $2x + 3y = 8$. Find i) means of the variables x and y ii) correlation coefficient between x and y. **(07 Marks)**

c. Three typists A, B, C typed 50%, 30% and 20% of pages of a book. The percentage of defectively typed pages by them is 3, 4, 5 respectively. If a page is selected from the book at random, what is the probability that it is defectively typed and it is typed by 'A'? **(07 Marks)**

6 a. The random variable X has the following probability mass function

X :	0	1	2	3	4	5
P(X) :	K	3K	5K	7K	9K	11K

i) find K ii) find $P(X < 3)$ iii) find $P(3 < X \leq 5)$. **(06 Marks)**

b. Alpha – particles are emitted by a radio active source at an average of 5 emissions in 20 minutes. What is the probability that there will be i) exactly two emissions ii) at least two emissions in 20 minutes? **(07 Marks)**

c. A sample of 100 dry battery cells tested to find the length of life produced by a company and following results are recorded : mean life = 12 hours, standard deviation = 3 hours. Assuming data to be normally distributed, find the expected life of a dry cell :
i) have more than 15 hours ii) between 10 and 14 hours. **(07 Marks)**

7 a. Explain the following terms : i) Null hypothesis ii) Standard error iii) Test of significance. **(06 Marks)**

b. Find the range of number of heads out of 64 tosses of a coin which will ensure fairness of coin at 5% level of significance using binomial distribution. **(07 Marks)**

c. A survey conducted on 64 families with 3 children each and recorded as follows :

No. of Male children :	0	1	2	3
No. of families :	6	19	29	10

Apply Chi – Square test to test whether male and female children are equiprobable at 5% level of significance. **(07 Marks)**

8 a. The Joint probability distribution of two Random variable X and Y are given as :

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

i) find Marginal distribution of X and Y ii) find COV(X,Y). **(06 Marks)**

b. Find the unique fixed probability vector of the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$ **(07 Marks)**

c. A player's luck follows a pattern. If he wins a game the probability of winning next game is 0.6. However if he loses the game the probability of losing the next game is 0.7. There is an even chance of winning the first game. If so i) what is the probability of winning 2nd game ii) What is the probability of winning 3rd game? **(07 Marks)**

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06ES42

Fourth Semester B.E. Degree Examination, June/July 2011
Microcontrollers

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Compare the following :
   i) Microprocessors and Microcontrollers
   ii) RISC and CISC architectures
   iii) Harvard and Von-Neumann architecture. **(12 Marks)**

b. Explain the following pins and its functions of 8051 microcontrollers:
   i) ALE ii) $\overline{PSEN}$ iii) $\overline{EA}$ iv) $\overline{RD}$ **(08 Marks)**

2 a. Explain the addressing modes of 8051 with examples. **(08 Marks)**

b. Explain the following instructions of 8051 with examples for each instruction:
   i) SUBB A, direct ii) PUSH direct iii) MOVE A, @A+DPTR **(06 Marks)**

c. Examine the following code and analyse the result with flag register content:
   MOV A, # +96
   MOV R_1, # +70
   ADD A, R_1 **(06 Marks)**

3 a. Classify the CALL instruction in 8051. Explain each one. **(06 Marks)**

b. What are the steps executed by the 8051 microcontroller when the following instructions is executed:
   i) RET ii) AJMP addr11 **(06 Marks)**

c. Write an ALP to add 'N' 8-bit numbers available from memory location START. Display the result at port 0 and port 1. **(08 Marks)**

4 a. Explain different data types in 8051 C. **(04 Marks)**

b. Write an 8051 C program to get a byte of data from P_1, wait ½ second and then send it to P_2. **(08 Marks)**

c. Write an 8051 C program to convert ASCII digits of any two values to packed BCD and display it on port 1. **(08 Marks)**

PART – B

5 a. Explain TMOD and TCON SFR registers of 8051 timers. **(08 Marks)**

b. Write an ALP or C program to generate a frequency of 100 Hz square wave, using timer 0 in mode 1. Assume crystal frequency = 11.0592 MHz. **(12 Marks)**

6 a. What is baud rate? Which timer of the 8051 is used to set the baud rate? **(03 Marks)**

b. Explain SCON register with its bit pattern. **(07 Marks)**

c. Write an 8051 program to send the date message "MORNING" of length seven characters at a baud rate of 4800, b-bit data, 1 stop bit serially. **(10 Marks)**

7 a. Compare polling interrupts. What are the steps microcontroller perform upon activation of an interrupt. **(06 Marks)**

b. How the interrupts in 8051 is classified? Explain each interrupt. **(06 Marks)**

c. Write a program using interrupts to get data from P_1 and send it to P_2 while Timer 1 is turning ON and OFF the LED connected to $P_{0.4}$ every second. **(08 Marks)**

8 a. Interface ADC 0804 to 8051 and write a program to read analog data and display the converted data at port 2. **(10 Marks)**

b. Show the interfacing of a stepper motor to 8051 and write a program to rotate stepper motor 5 steps in clockwise direction and 10 steps in anticlockwise direction with a delay between each step. **(10 Marks)**

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06ES43

Fourth Semester B.E. Degree Examination, June/July 2011
Control Systems

Time: 3 hrs. Max. Marks:100

Note: *1. Answer FIVE full questions selecting at least TWO questions from each part.*
2. Missing data, if any may be suitably assumed.

PART – A

1 a. Distinguish closed loop control system from open loop control system with suitable examples. **(06 Marks)**

b. For the circuit shown in Fig. Q1 (b).
i) Draw the mechanical network
ii) Write the differential equation describing the system.
iii) Draw the force-voltage analogous electrical circuit after writing the corresponding electrical equation. **(10 Marks)**

c. Obtain the transfer function of the system shown in Fig. Q1 (c). **(04 Marks)**

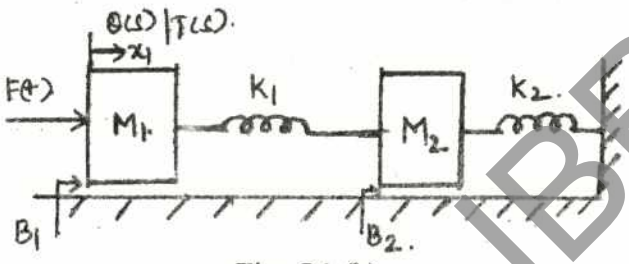

Fig. Q1 (b)

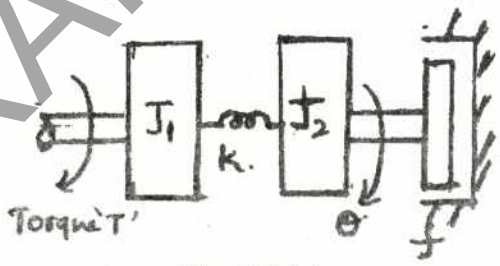

Fig. Q1 (c)

2 a. Explain the block diagram rule regarding:
i) Combining block in cascade.
ii) Moving a slimming point after a block.
iii) Moving a take off point beyond a block. **(06 Marks)**

b. Reduce using block diagram reduction technique the circuit shown in Fig. Q2 (b) and hence find its transfer function C(s)/ R(s).

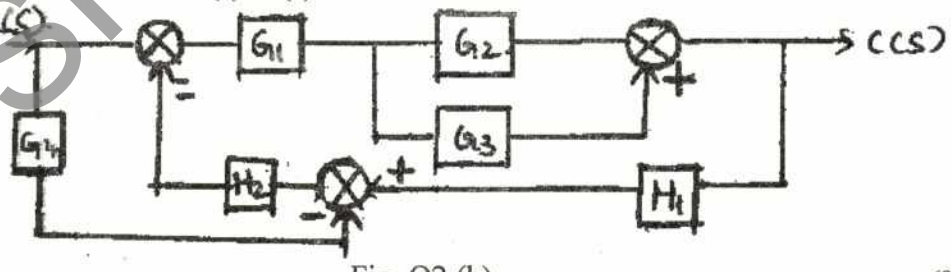

Fig. Q2 (b) **(08 Marks)**

c. For the network shown in Fig. Q2 ©construct the signal flow graph.

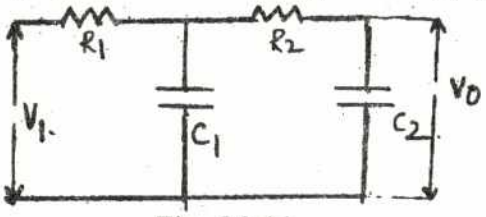

Fig. Q2 (c) **(06 Marks)**

3 a. Explain the following time domain specification of a second order system:
i) Rise time ii) Delay time iii) Peak time iv) Maximum over shoot. **(06 Marks)**

b. A signal is represented by the equation $\frac{d^2\theta}{dt^2}+10\frac{d\theta}{dt}=150e$ where e = (r - θ) is the actuating signal. Calculate the value of damping ratio, undamped and damped frequency of oscillation. Also draw the block diagram of the system and find its closed loop transfer function. **(08 Marks)**

c. For a unity feed back system $G(s)=\frac{s(s+1)}{s^2(s+3)(s+10)}$. Determine the type of the system, error co-efficient and steady state error for input $r(t)=1+3t+\frac{t^2}{2}$.. **(06 Marks)**

4 a. State R-H criterion and discuss its limitation. **(06 Marks)**

b. The open loop transfer function of a unity feed back system is given by $G(s)=\frac{k}{s(s+3)(s^2+s+1)}$. Find the value of k that will cause sustained oscillation and hence find the oscillation frequency. **(10 Marks)**

c. Test the stability of the system characterized by its characteristics equation $s^4+2s^3+3s^2+4s+5=0$. **(04 Marks)**

PART – B

5 a. State the different rules for construction of root loci. **(08 Marks)**

b. Sketch the root locus for a negative feedback control system given by $G(s)H(s)=\frac{k(s+a)}{s(s+1)(s+2)}$. **(12 Marks)**

6 a. Explain Nyquist stability criterion. **(06 Marks)**

b. For the given system $G(s)H(s)=\frac{k(s+a)}{s(s-1)}$ sketch the Nyquist plot and determine whether the system is stable or unstable. **(14 Marks)**

7 a. Define the following terms:
i) Gain cross over frequency ii) Phase cross over frequency
iii) Gain margin iv) Phase margin **(06 Marks)**

b. Construct the Bode plot for a unity feedback control system with $G(s)=\frac{10(s+10)}{s(s+2)(s+5)}$. Find its gain margin and phase margin. Comment on the stability. **(14 Marks)**

8 a. List the properties of state Transition matrix. **(04 Marks)**

b. Consider a system given by $\dddot{y}+9\ddot{y}+26\dot{y}+24y=6u$. Obtain its state model. **(06 Marks)**

c. Obtain the state transition matrix Q (t) of the following system.
$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$. Also obtain the inverse of the state transition matrix $\phi^{-1}(t)$. **(10 Marks)**

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06EC44

Fourth Semester B.E. Degree Examination, June/July 2011

Signals and Systems

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find and sketch the following signals and their derivatives:
i) $x(t) = u(t) - u(t-a)$; $a > 0$ ii) $y(t) = t[u(t) - u(t-a)]$; $a > 0$. **(06 Marks)**

b. Given the signal $x[n] = (8-n)\{u[n] - u[n-8]\}$, determine and sketch :
i) $y_1[n] = x[4-n]$ ii) $y_2[n] = x[2n-3]$. **(04 Marks)**

c. Determine whether the following signals are energy or power signals. Find the corresponding energy or power associated with the signal.
i) $x[n] = (1/4)^n u[n]$ ii) $x[n] = u[n]$ **(04 Marks)**

d. Fig.Q1(d)(i) shows a staircase like signal x(t) that may be viewed as a superposition of four rectangular pulses. Starting with the rectangular pulse shown in Fig.Q1(d)(ii), construct the waveform and express x(t) in terms of g(t). **(06 Marks)**

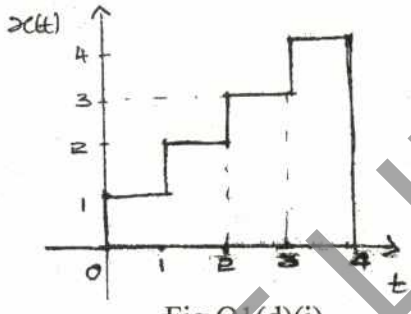

Fig.Q1(d)(i)

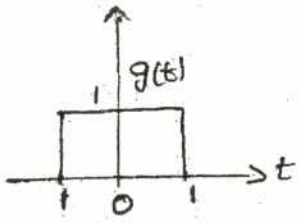

Fig.Q1(d)(ii)

2 a. Show that : i) $x(t) * h(t) = h(t) * x(t)$
ii) $x[n] * h_1[n] * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$ **(06 Marks)**

b. Given $x(t) = u(t) - u(t-3)$ and $h(t) = u(t) - u(t-2)$, evaluate and sketch $y(t) = x(t) * h(t)$. **(06 Marks)**

c. A LTI system has the impulse response given by $h[n] = u[n] - u[n-10]$. Determine the output of the system when the input is $x[n] = u[n-2] - u[n-7]$ using the convolution sum. Show the details of your computation. Sketch all the sequence. **(08 Marks)**

3 a. A discrete LTI system is characterized by the unit sample response $h[n] = \frac{1}{2}\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2]$. Determine :

i) Frequency response $H(e^{j\Omega})$ and plot the magnitude component

ii) Steady state response of the system for the input $x[n] = 5\cos\frac{\pi n}{4}$

iii) Total response of the system for the input x[n] = u[n] assuming that the system is initially at rest. **(10 Marks)**

b. Determine whether the system described by the following are stable or causal:
i) $h[n] = (1/2)^n u[n]$ ii) $h(t) = e^t u(-1-t)$ **(06 Marks)**

3 c. Determine the differential equation representation for the block diagram shown in Fig.Q3(c). **(04 Marks)**

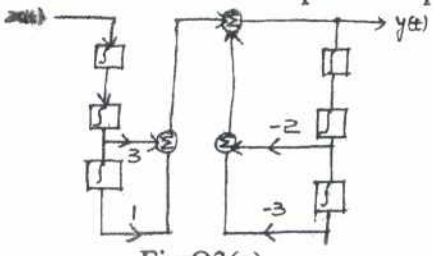

Fig.Q3(c)

Fig.Q4(b)

4 a. Evaluate the DTFS representation for the signal $x[n]=\sin\left(\frac{4\pi}{21}n\right)+\cos\left(\frac{10\pi}{21}n\right)+1$. Sketch the magnitude and phase spectra. **(08 Marks)**

b. Find the exponential Fourier series of the waveform shown in Fig.Q4(b). **(08 Marks)**

c. Explain the orthogonality of complex sinusoidal signals. **(04 Marks)**

PART – B

5 a. Find the DTFT of the signal $x[n]=n\left(\frac{1}{2}\right)^{|n|}$. **(07 Marks)**

b. Determine the signal x[n] if its spectrum is shown in Fig.Q5(b). **(07 Marks)**

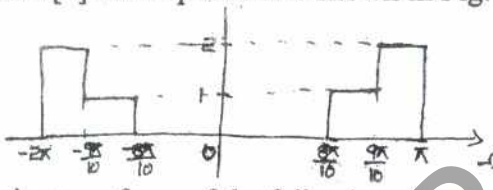

Fig.Q5(b)

c. Determine the Fourier transform of the following signals :

i) $x(t)=e^{-3t}u(t-1)$ ii) $x(t)=e^{-a|t|}$ **(06 Marks)**

6 a. Find the frequency response and impulse response of the system described by the differential equation : $\frac{d^2}{dt^2}y(t)+3\frac{dy(t)}{dt}+2y(t)=2\frac{d}{dt}x(t)+x(t)$. **(08 Marks)**

b. The output of a system in response to an input $x(t)=e^{-2t}u(t)$ is $y(t)=e^{-t}u(t)$. Find the frequency response and the impulse response of this system. **(08 Marks)**

c. Obtain an expression for the Fourier transform in terms of DTFT. **(04 Marks)**

7 a. Find the z-transform of the following and indicate the region of convergence : **(12 Marks)**

i) $x[n]=\alpha^{|n|}$; $0<|\alpha|<1$; ii) $x[n]=2^n\sin\Omega_0(n-2)u(n-2)$; iii) $x[n]=n(n-1)a^nu[n]$

b. Find the inverse z transform of the following :

i) $X(z)=\frac{z^2+z^2}{z^2-\frac{3}{4}z+\frac{1}{8}}$; $|z|>\frac{1}{2}$ ii) $X(z)=\frac{1-az^{-1}}{z^{-1}-a}$; $|z|>\frac{1}{a}$ **(08 Marks)**

8 a. A discrete LTI system is characterized by the difference equation $y[n]=y[n-1]+y[n-2]+x[n-1]$. Find the system function H(z). Plot the poles and zeros of H(z) and indicate the ROC if the system is (i) stable, (ii) causal. Also determine the unit sample response of the stable system. **(09 Marks)**

b. Solve the following difference equation using the unilateral z transform :
$x[n-2]-9x[n-1]+18x[n]=0$ with the initial conditions $x[-1]=1$ and $x[-2]=9$. **(07 Marks)**

c. A system is described by the difference equation :

$$y[n]-y[n-1]+\frac{1}{4}y[n-2]=x[n]+\frac{1}{4}x[n-1]-\frac{1}{8}x[n-2].$$

Find the transfer function of the inverse system. Does a stable and causal inverse system exist? **(04 Marks)**

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06EC45

Fourth Semester B.E. Degree Examination, June/July 2011
Fundamentals of HDL

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions selecting atleast TWO questions from each part.***

PART - A

1 a. Mention different types of description in HDL. Explain structural and mixed type of description with example. **(10 Marks)**

b. Explain Verilog data types. **(05 Marks)**

c. Discuss the major differences between VHDL and Verilog. **(05 Marks)**

2 a. Write a VHDL and Verilog code for a 2 × 1 multiplexer in data flow description using signal assignment statement. **(08 Marks)**

b. Write a VHDL code in data flow description for a 2 – bit magnitude comparator with the help of truth table and simplified Boolean expressions. **(12 Marks)**

3 a. Explain CASE statement with syntax. Write a behavioural description of a positive edge-triggered JK flipflop using CASE statement in VHDL and Verilog code. **(10 Marks)**

b. Explain For – loop and while – loop statements in VHDL and Verilog. **(06 Marks)**

c. Write a Verilog code for calculating the factorial of positive integers using while loop. **(04 Marks)**

4 a. What is binding? Discuss binding between a library and component in VHDL. **(06 Marks)**

b. Write the structural description of a D – Latch using VHDL and Verilog code. **(08 Marks)**

c. Briefly explain Generate, Generic and Parameter statements with an example. **(06 Marks)**

PART - B

5 a. Explain the use of procedure in VHDL and task in Verilog with description of an N – bit ripple carry adder. **(10 Marks)**

b. Write a VHDL function to find the larger of two signed numbers. **(06 Marks)**

c. Write a note on Verilog file processing. **(04 Marks)**

6 a. Explain with syntax VHDL package and package body. **(04 Marks)**

b. Write a VHDL code for addition of two 5 × 5 matrices using a package. **(08 Marks)**

c. With a block diagram and function table of SRAM, write a Verilog code for 16 × 8 SRAM. **(08 Marks)**

7 a. Write a mixed Language description of a full adder invoking a VHDL entity from a Verilog module. **(10 Marks)**

b. Write a mixed language description of an AND gate invoking Verilog module from VHDL module. **(06 Marks)**

c. What are the limitations of mixed language description? **(04 Marks)**

8 a. Define synthesis. With flow chart, explain the steps involved in synthesis process. **(08 Marks)**

b. Write VHDL code for signal assignment statement Y = 2 * X + 3. Show the synthesized logic symbol and gate level diagram. Write structural code in Verilog using gate level diagram. **(12 Marks)**

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06EC46

Fourth Semester B.E. Degree Examination, June/July 2011
Linear ICs and Applications

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain common mode voltage, common mode voltage gain and common mode rejection ratio for operational amplifiers. Show that $V_{0(cm)} = \frac{V_{i(cm)}}{CMRR} \times A_v$ **(10 Marks)**

b. With a neat circuit diagram, explain direct-coupled non-inverting amplifier with necessary design steps. **(05 Marks)**

c. Sketch an op-amp difference amplifier circuit. Derive an equation for the output voltage and explain the operation. **(05 Marks)**

2 a. Explain the operation of a high input impedance capacitor-coupled non-inverting amplifier with a neat circuit diagram. **(10 Marks)**

b. A capacitor-coupled non-inverting amplifier is to have a +24V supply, a voltage gain of 100, an output amplitude of 5V, a lower cutoff frequency of 75 Hz and a minimum load resistance of 5.6 kΩ. Using a 741 op-amp, design a suitable circuit. Given: $I_{B(max)}$= 500 nA, f_l = 75 Hz, R_L = 5.6 kΩ **(10 Marks)**

3 a. Discuss the different frequency compensating methods with the circuit diagrams. Also show how each one of them affect the frequency response of an op-amp. **(12 Marks)**

b. Calculate the slow-rate limited cutoff frequency for a voltage follower circuit using a 741 op-amp, if the peak of sinewave output is to be 5 V. Determine the maximum peak value of the sinusoidal output voltage that will allow the 741 voltage follower circuit to operate at 800 kHz unity-gain cutoff frequency. Given : S = 0.5 V/μs. **(05 Marks)**

c. Calculate the cut-off frequency-limited risetime for a voltage follower circuit using a 741 op-amp. Also, determine the slow rate-limited risetime if the output amplitude is to be 5V. Given: S = 0.5 V/μs, f_2 = 800 kHz. **(03 Marks)**

4 a. Show how two op-amp dead zone circuits can be combined with a summing circuit to produce precision limiting on both positive and negative half-cycles of the output waveform. Draw the voltage waveforms through out the circuit and explain its operation. **(10 Marks)**

b. Design a precision voltage source to provide an output of 9 V. The available supply is ±12V. Allow for approximately ±10% tolerance on the zener diode voltage. Use 741 op-amp. Given : $I_{B(max)}$ = 500 nA, I_z = 20 mA. **(10 Marks)**

PART – B

5 a. With a neat circuit diagram, explain the operation of a precision rectifier peak detector circuit. Draw the input and output waveforms. Write the equation for calculating the capacitor value. **(08 Marks)**

b. With a neat circuit diagram, explain the operation of a Log amplifier using op-amp. Derive the output voltage equation. **(06 Marks)**

c. Using a 741 op-amp, with a supply of $\pm$12V, design a phase-shift oscillator to have an output frequency of 3.5 kHz. Given $I_{B(max)} = 500$ nA. **(06 Marks)**

6 a. With neat circuit diagrams, explain how diodes may be used to select the trigger points of an inverting Schmitt trigger circuit. **(06 Marks)**

b. With a neat circuit diagram and waveforms, explain the operation of op-amp based astable multivibration. Write the design steps. **(08 Marks)**

c. Design a second-order low-pass filter circuit to have a cutoff frequency of 1 kHz. **(06 Marks)**

7 a. Define the following performance parameters of a voltage regulator;
Line regulation, Load regulation, Ripple rejection. **(06 Marks)**

b. With a neat functional diagram, explain the operation of a low-voltage regulator using IC723. **(08 Marks)**

c. Bring out the limitations of Linear voltage regulators. **(06 Marks)**

8 a. Explain the working of monostable multivibrator using 555 timer with a neat functional diagram and waveforms. Derive the equation for its pulse width. **(08 Marks)**

b. Draw the block diagram representation of PLL and explain. **(06 Marks)**

c. With a neat circuit diagram and staircase waveform, explain the operation of counter type Analog-to-Digital converter. **(06 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2011
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. Show that the angle between the lines whose direction ratios are 2, 1, 1 and 4, $\sqrt{3}-1$, $-\sqrt{3}-1$ is 60°. **(07 Marks)**

c. Find the value of K such that the set of four points (0, -1, -1) (-4, 4, 4) (k, 5, 1) and (3, 9, 4) are co-planar. **(07 Marks)**

2 a. Derive the equation of the plane in the intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(06 Marks)**

b. Find the equation of the plane which passes through the point (3, -3, 1) and is perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. **(07 Marks)**

c. Show that the lines : $\frac{x+3}{2}=\frac{y+5}{3}=\frac{z-7}{-3}$ and $\frac{x+1}{4}=\frac{y+1}{5}=\frac{z+1}{-1}$ are coplanar and hence find the equation of the plane in which they lie. **(07 Marks)**

3 a. Find a unit vector perpendicular to both the vectors $\vec{A}=2\hat{i}+\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}-\hat{j}+2\hat{k}$. **(06 Marks)**

b. If $\vec{a},\vec{b},\vec{c}$ are any three vectors, prove that :

i) $[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}]=2[\vec{a},\vec{b},\vec{c}]$

ii) $[\vec{b}\times\vec{c},\vec{c}\times\vec{a},\vec{a}\times\vec{b}]=[\vec{a},\vec{b},\vec{c}]^2$ **(07 Marks)**

c. Find the value of λ so that the vectors $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}$ $\vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=\hat{j}+\lambda\hat{k}$ are coplanar. **(07 Marks)**

4 a. A particle moves along a curve $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$ where t is the time variable. Determine its velocity and acceleration vectors and also the magnitudes of velocity and acceleration at $t = 2$. **(06 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at the point (2, -1, 2). **(07 Marks)**

c. Find the directional derivative of $\phi = xy^2 + yz^3$ at (2, -1, 1) in the direction of vector $\hat{i}+2\hat{j}+2\hat{k}$. **(07 Marks)**

5 a. Find the divergence and curl of the vector $\vec{F}=(3x^2y-z)\hat{i}+(xz^3+y^4)\hat{j}-(2x^3z^2)\hat{k}$. **(06 Marks)**

b. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ show that i) $\nabla.\vec{r}=3$; ii) $\nabla\times\vec{r}=0$. **(07 Marks)**

c. Find the constants a, b, c, such that the vector field $\vec{f}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$ is irrotational. **(07 Marks)**

6 Find :

a. $L(4\sinh 5t - 5\cos 4t)$ **(05 Marks)**

b. $L(\cos at \cos bt)$ **(05 Marks)**

c. $L(e^{-t}\cos^2 t)$ **(05 Marks)**

d. $L(te^{-t}\sin t)$ **(05 Marks)**

7 Find :

a. $L^{-1}\left[\frac{1}{s+3}+\frac{3}{2s+7}-\frac{5}{3s-z}\right]$ **(05 Marks)**

b. $L^{-1}\left[\frac{2s+1}{(s-2)(s-3)}\right]$ **(05 Marks)**

c. $L^{-1}\left[\frac{s}{s^2+6s+13}\right]$ **(05 Marks)**

d. $L^{-1}\left[\log\left(\frac{s+1}{s-1}\right)\right]$ **(05 Marks)**

8 a. Using Laplace transform method solve, $\frac{d^2y}{dt^2}+\frac{3dy}{dt}+2y=0$ under the conditions $y(0)=1, y'(0)=0$. **(10 Marks)**

b. Solve by using Laplace transforms $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$, $x=1$, $y=0$ at $t=0$. **(10 Marks)**

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